Chapter 4
"An Analytical Approach to Investments, Finance and Credit"

RISK, RETURN, MARKET AND OTHER PORTFOLIO COMPARATIVE ANALYSIS

Sharpe Ratio, CAPM, Jensen's Alpha, Treynor Measure, and M-Square

Portfolio Management Performance Measurement: An Overview

- There are many ways to measure the performance of a portfolio.
- The most common method is to compare the performance against last year or against last few years, often called trend analysis.
- Investors will always ask how the portfolio returns are performing from year to year.
- However, displaying the trend analysis to an investor who is in the process of deciding whether to come into the fund is not enough.
- The investor will demand a comparative analysis of portfolio performance against other portfolios, or against other asset classes, or the market.
- These comparative measures include ratios such as the Sharpe ratio, CAPM, Jensen's Alpha, and Treynor measure—all these are ratios that give the investor comfort about how the portfolio performed against the market or other assets.

Portfolio Efficiency to Optimization

From the point of efficiency, the portfolio manager is seeking to achieve an even a higher return delta (rate of change), but as discussed, it will also come with higher risk. The optimization point is where the additional return, or the rate of change going from bonds to stocks, is lower than the rate of change of additional risk—basically, higher return delta at a lower risk delta. That achievement is called the optimum point and is the basis of the Sharpe ratio. Figure 4.3 shows a basic illustration of the optimization process.

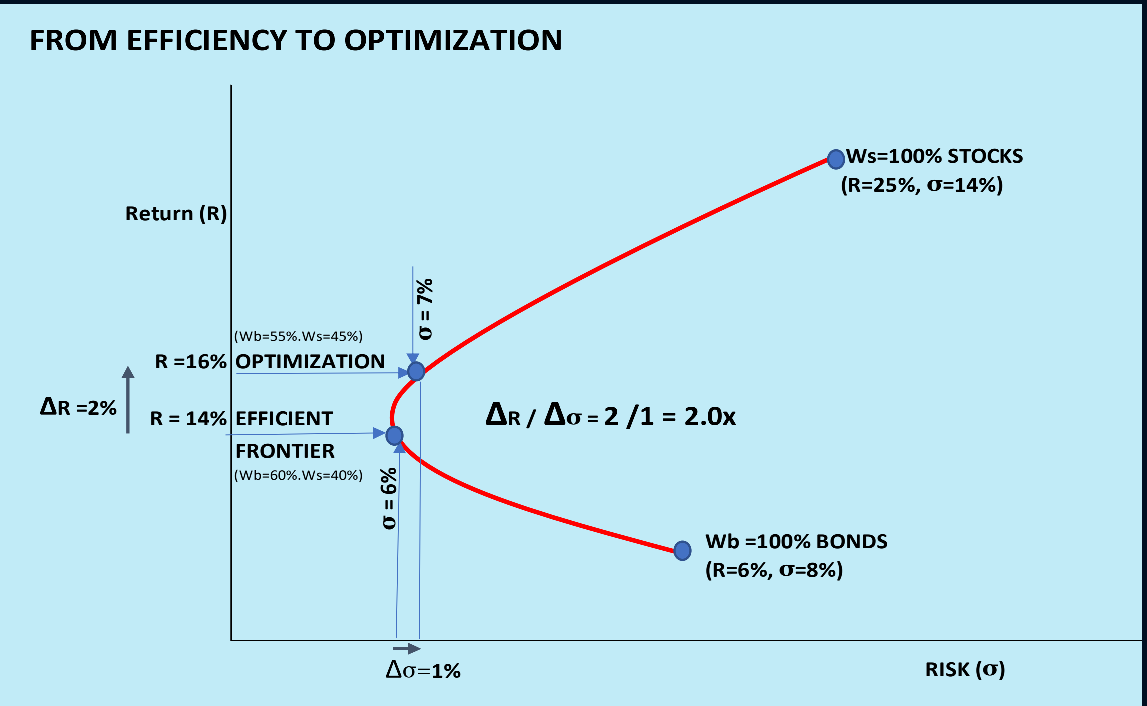

Fig. 4.3

PORTFOLIO PERFORMANCE RATIOS

- **Sharpe Ratio (Optimization Point)**

The Sharpe ratio, in its basic form, is the relationship between return (numerator) and risk (denominator). The numerator is adjusted to reflect the risk premium, done by taking the absolute rate of return and subtracting the risk-free rate, which, by definition, has little to no risk for the given period. The denominator representing the risk is measured by the volatility of the investment return or the standard deviation of the asset class for the same time period. The Sharpe ratio formula is as follows:

$$SR = \frac{R_P - Rf}{\sigma_P}$$

where, R_P is the absolute return of a given portfolio, Rf is the risk-free rate, and σ_P is the standard deviation of the portfolio.

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EXAMPLE OF COMPARING TWO PORTFOLIOS AND THE MARKET: Figure 4.4 below compares the measurements portfolio Z to portfolio X and as compared to the stock market benchmark.

Portfolio Perfomance Ratio Analysis

	Description	Symbol	Calculation	Stock Portfolio Z	Stock Portfolio X	Stock Benchmark Market (m)
INPUT	Average Return	*R*		19.00%	20.00%	10.00%
	Risk Free Return	*Rf*		2.00%	2.00%	2.00%
	Standard Deviation	*σ*		26.00%	36.00%	18.00%
	Beta	*β*		1.300x	1.500x	1.000x
OUTPUT	Risk Premium Return	*RPR*	*R - Rf*	17.00%	18.00%	8.00%
	Market Premium	*Pm*	*Rm - Rf*	8.00%	8.00%	8.00%
	Capital Asset Pricing Model	*CAPM*	*Rfr + β. Pm*	12.40%	14.00%	10.00%
	Sharpe Ratio	*SR*	*RPR / σ*	0.654	0.500	0.444
	Jensen's Alpha	*α*	*R - CAPM*	6.600%	6.000%	0.000%
	Treynor Measure	*T*	*RPR / β*	13.077%	12.000%	8.000%
	M-Square	M^2	*((σm/σp)*P)+Rf*	13.769%	11.000%	10.000%

Figure 4.4

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From Portfolio Efficiency to Optimization – Example:

Assuming a 3.0% risk-free rate that has a 0% standard deviation, the Sharpe ratio is calculated as follows:

$$at\ optimization\ point,\quad SR = \frac{R_P - Rf}{\sigma_P} = \frac{16-3}{7} = 1.86$$

The calculation shows that every 1.86% increase in return comes with an equivalent 1% of risk. The optimization point is the point with the highest possible Sharpe ratio. Using the same illustration (figure 4.3) to calculate the Sharpe ratio using the efficient frontier risk and return points, the ratio calculates as follows:

$$at\ efficiency\ point, SR = \frac{R_P - Rf}{\sigma_P} = \frac{14-3}{6} = 1.83$$

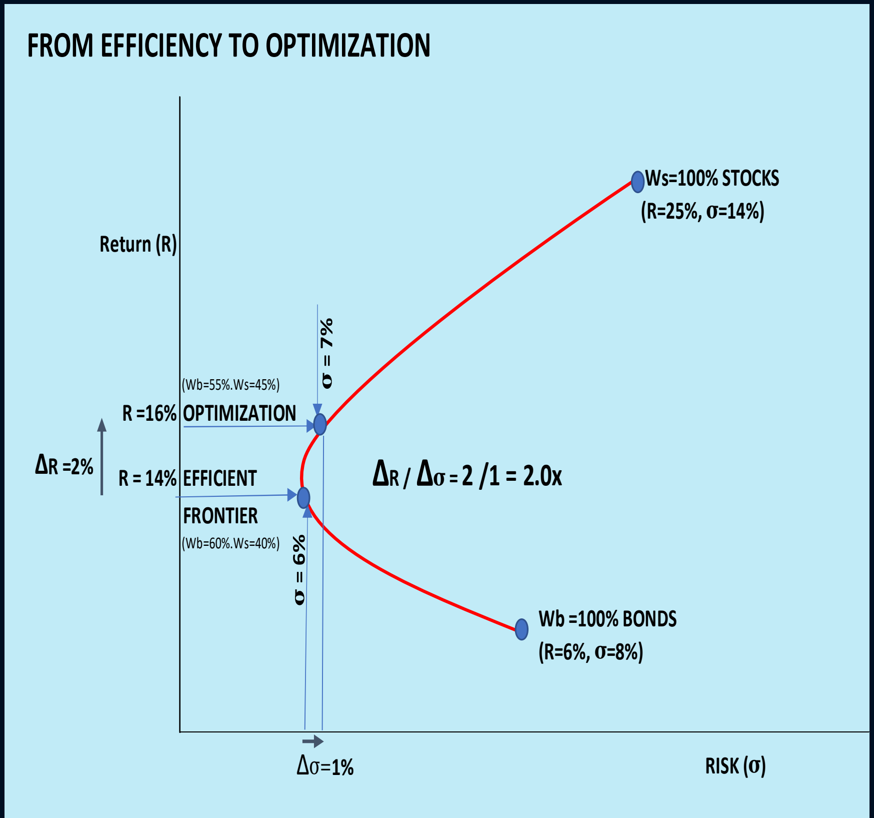

Fig. 4.3

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Capital Asset Pricing Model (CAPM)

- **The CAPM is a formula that was developed to calculate the expected return of any risky asset class ($\mathbf{ER_i}$) as compared to the systematic market risk.** This formula will be used extensively in later chapters, not only for portfolio management applications, but also as a discount rate for determining the present value of the equity invested in a firm. The formula is as follows:

$$ER_i = R_f + \beta\left(ER_m - R_f\right)$$

where, R_f is the risk-free rate, β is the beta, and ER_m is the expected market return.

- **CAPM is used as the basis for the minimum expectation of an investor who is seeking to evaluate a portfolio of stocks or a single stock adjusted to market fluctuations.**

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Capital Asset Pricing Model (CAPM)

The objective of CAPM is set as the basis for evaluating if the stock is fairly valued as compared to the market.

- For example, let's assume that the investor is looking to buy XYZ Inc.'s stock that has a **beta (β) of 1.5x**, which means that the volatility of such stock is 1.5x the volatility of the total equity market index. If the **market is anticipated to grow 10% this year**, then the return of such investment should grow at 15% (1.5 x 10%). The CAPM formula adjusts for risk-free rate after establishing the market risk premium return $(ER_m - R_f)$ or (10% -R_f), so the expected risk premium return for such stock is 1.5x the market premium. **Assuming the risk-free rate is 2%,** the expected investment return is calculated at 14% as follows:

$$ER_i = R_f + \beta(ER_m - R_f) = 2\% + 1.5\ (10\% - 2\%) = 2\% + 12\% = 14\%$$

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Jensen's Alpha Ratio

Jensen's alpha, developed by mutual fund manager Michael Jensen in the late 1960s, **is a formula that determines the average return over (positive alpha) or below (negative alpha) the expectation calculated by the CAPM.** As mentioned previously, CAPM represents the minimum expected return of a portfolio or single stock adjusted to the market expectation. Jensen's alpha, or simply alpha (α), if positive, represents the excess return over CAPM. The formula is as follows:

$$\alpha z = R_i - [R_f + \beta(R_m - R_f)] \ or \ \alpha = R_i - CAPM$$

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Jensen's Alpha Ratio

Using the 20% example, let's assume portfolio Z had beta (βz) of 1.3 and portfolio X had beta (βx) of 1.5. Let's assume the overall market returned 10% and risk-free rate is 2.0% for that period. The following formulas calculates the alphas for portfolio Z (αz) and portfolio X (αx)

$$\alpha z = R_i - [R_f + \beta(R_m - R_f)] = 20\% - [2\% + 1.3\ (10\% - 2\%) = 7.6\%$$
$$\alpha z = 7.6\%$$

$$\alpha x = R_i - [R_f + \beta(R_m - R_f)] = 20\% - [2\% + 1.5\ (10\% - 2\%) = 6.0\%$$
$$\alpha x = 6.0\%$$

The alpha for portfolio Z at 7.6% is higher than portfolio X of 6.0%, demonstrating that despite both having beat the market showing positive alpha (α), portfolio Z had a better performance when adjusting for risk which is determined by the lower beta (β). Let's use another example where portfolio Z shows 19% return and portfolio X shows 20%

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Treynor Ratio

The Treynor Ratio focuses on the relationship between the portfolio risk premium return and the beta (β) of the portfolio. It is expressed in factors (positive or negative) or as a multiple of the market premium risk. The formula is as follows:

$$T = \frac{R_P - Rf}{\beta_P}$$

The Treynor ratio, also known as the reward-to-volatility ratio, is designed to assess the portfolio's performance against the benchmark. Instead of measuring a portfolio return only against the risk-free rate, the ratio examines how well a portfolio outperforms the market.

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Treynor Ratio

Using the previous example, let's assume the market benchmark had a 10% return which represents beta (β=1), portfolio Z and portfolio X returned 19% and 20% respectively. Portfolios Z and X had betas (β) of 1.30 and 1.50, respectively. Also, let's assume the risk-free rate (treasury bills) is 2.0%. The Treynor value of each is calculated as follows:

- Market $Tm = \frac{R_m - Rf}{\beta_m} = \frac{.10-.02}{1} = .080 = 8.000\%$
- Portfolio Z $Tz = \frac{R_z - Rf}{\beta_z} = \frac{.19-.02}{1.3} = .1307 = 13.077\%$
- Portfolio X $Tx = \frac{R_x - Rf}{\beta_x} = \frac{.20-.02}{1.5} = .120 = 12.000\%$

The higher the Treynor ratio (T), the more efficient the portfolio. Like the Sharpe Ratio discussed above, if the analyst was only evaluating the portfolio on return performance alone, he or she may have recognized that portfolio X have returned the best results. The Treynor ratio is adjusted to reflect the risk adjusted to the market

PORTFOLIO PERFORMANCE RATIOS

M Squared Ratio

M^2 measures the difference between the excess return of the portfolio over the market. Unlike the Sharpe ratio that is measured in units of return versus risk, M^2 is expressed as a percentage return making it easier for the investor to read when analyzing a portfolio. M^2 is one of the newest modern portfolio measurement methods only developed in 1997 by the Nobel prize winner Franco Modigliani and his granddaughter, Leah Modigliani, hence the concept of M squared. The formula is as follows:

$$M^2 = SR \cdot \sigma_m + R_f$$

Where, SR is the Sharpe ratio of the risky portfolio, σ_m is the market benchmark standard deviation and R_f is the risk- free rate. The M^2 ratio can be written also as follows:

$$M^2 = \frac{R_p - R_f}{\sigma_p} \cdot \sigma_m + R_f \text{ or } M^2 = RPRp \,.\, \frac{\sigma_m}{\sigma_P} + R_f$$

Where is RPR_p is the portfolio risk premium return, σ_p is the portfolio's standard deviation, σ_m is the market benchmark standard deviation and R_f is the risk- free rate.

PORTFOLIO PERFORMANCE RATIOS

M Squared Ratio

Let's use the same information used above to compare portfolio Z to portfolio X in order to measure their M^2s. Let's assume portfolio Z had returns of 19% with standard deviation of 26% and portfolio X had returns of 20% with standard deviation of 36%. The market benchmark had return of 10% with standard deviation of 18%. The risk-free rate had a return of 2.0%. The M^2s for these portfolios are calculated as follows:

$$M_Z^2 = (Rz - Rf)\,.\frac{\sigma_m}{\sigma_z} + R_f = (19\% - 2\%)\,\frac{18\%}{26\%} + 2\% = 13.769\%$$

$$M_x^2 = (Rx - Rf)\,.\frac{\sigma_m}{\sigma_x} + R_f = (20\% - 2\%)\,\frac{18\%}{36\%} + 2\% = 11.000\%$$

From the calculations above the analyst can conclude that despite the absolute return for portfolio X of 20% is higher than portfolio Z's 19.0%, portfolio Z has a significant higher M^2 of 13.769% versus 11.00%.

PORTFOLIO PERFORMANCE RATIOS
Other Useful Portfolio Analysis Ratios

Burke Ratio (based on Drawdowns instead of Standard Deviation)

The Burke ratio, also referred to as "a sharper Sharpe ratio", is similar to the Sharpe Ratio in that it also measures the risk-adjusted performance of the portfolio. It uses the same numerator of Rp – Rf, but instead of using the portfolio's standard deviation as the denominator, the Burke ratio uses the concept of drawdowns.

$$BR = \frac{Rp - R_f}{\dot{D}^2}$$

Where, Rp is the portfolio return, R_f is the risk-free return and D is the drawdown.

PORTFOLIO PERFORMANCE RATIOS

Other Useful Portfolio Analysis Ratios

Omega Ratio (based on Min/Max Variance instead of Standard Deviation)

This ratio is similar to the Sharpe ratio in that it compares the return to volatility. The Omega ratio, however, uses the higher points of distribution, arguing that these points could show a better assessment of volatility. It is done to combat the tendency of a distribution to be asymmetric with tail risk or negative skewness. The formula is a little complex but is written as follows:

$$\Omega = \frac{\int_{ERp}^{\infty}\big(1 + F(x)\big)dx}{\int_{-\infty}^{ERp} F\,(x)dx}$$

Where, F(x) is the cumulative probability distribution function of the returns

PORTFOLIO PERFORMANCE RATIOS
Other Useful Portfolio Analysis Ratios

- **Sortino Ratio (based on Loses instead of Standard Deviation)**

  The Sortino Ratio is a ratio that adjusts for trading losses. Using the standard deviation, the basis of calculating the Sharpe ratio, the measure is penalized by both downside and upside volatility. Sometimes investors will like to show the sudden uptick in their portfolio based on decisions they've made and using the Sharpe ratio might not make this as obvious as one might desire. This is due to the fact that it is offset by the series of historical downsides and upsides equally. The Sortino Ratio on the other hand only penalizes downside risk. The formula is as follows:

$$SoR = \frac{ERp - Rf}{\sigma_d}$$

  Where, ERp is the expected return of the portfolio, Rf is the risk-free rate and σ_d is the standard deviation of negative asset returns (downside risk). The downside risk is calculated as follows:

$$\sigma_d = \sqrt{\frac{\sum\left(R_\rho - ER_p\right)^2 f(t)}{n}}$$

  Where R_ρ are the historical returns of the portfolio, ER_p is Expected Return (threshold), n is the number of years or observations, $f(t)$ represents the arguments that are tested if the returns are higher or lower than the expected return (ER_p) . For example, $f(t)$ = 1 when total returns are higher than the target return (ER_p) and $f(t)$ = 0 if the historical returns are equal to zero or higher than the target return $of\ (ER_p)$.