Chapter 2
"An Analytical Approach to
Investments, Finance and Credit"

RISK, RETURN, TIME AND ALLOCATION

Covariance, Correlation and Efficient Frontiers

CHAPTER 1 REVIEW:
Measuring Return and Return Expectation

- Before you invest your money in any securities or any businesses, it's extremely important to consider and must measure the following four factors:
  1. **Return expectation**
  2. **Risk**
  3. **Allocation**
  4. **Time**

REVIEW: Measuring Return and Return Expectation

Scenario Analysis Method

S T O C K S

SCENARIO PERFROMANCE ANALYSIS

Scenario (s)	Probability (p)	ROR % (r_s)	p * r_s %	Deviation for Exp. Ret. (Dev.)	Square Deviation (SD) Dev^2	p * SD
		Stocks (s)				
Recession (Sr)	**25.0%**	**-12.00**	-3.00	-23.70	561.69	140.42
Normal (Sn)	**45.0%**	**14.00**	6.30	2.30	5.29	2.38
Boom (Sb)	**30.0%**	**28.00**	8.40	16.30	265.69	79.71
	100.0%		**11.70** %		**Variance=**	**222.51**
					SD =	**14.92** %

Figure 1.3

B O N D S

SCENARIO PERFORMANCE ANALYSIS

Economic Scenario (s)	Probability (p)	ROR % (r_b)	p * r_b %	Deviation for Exp. Ret. (Dev.)	Square Deviation (SD) Dev^2	p * SD
		Bonds (b)				
Recession	**25.0%**	**14.00**	3.50	9.75	95.06	23.77
Normal	**45.0%**	**5.00**	2.25	0.75	0.56	0.25
Boom	**30.0%**	**-5.00**	-1.50	-9.25	85.56	25.67
	100.0%		**4.25** %		**Variance=**	**49.69**
					SD =	**7.05** %

Figure 1.4

CHAPTER 1 REVIEW:

Measuring Return and Return Expectation

Return, Return Expectation, Risk and Allocation

Return:

Then the combined portfolio shown in figure 1.5 consisting of 60% stock and 40% bonds shows an expected combined return, variance, and standard deviation of 8.72%, 38.99% or .39x, and 6.24%, respectively. As expected, as we moved from the stock portfolio of 100% to a portfolio of 60% stock and 40% bonds, the return is calculated at 8.72% measured as

(Ws . Rs) + (Wb . Rb) = = (.60) (11.70%) + (.40) (4.25%) = 7.02% + 1.7% = 8.72%

Risk:

The Risk is measured by the amount of volatility needed to achieve the expected returns. The volatility is basically the variance and standard deviation of the historical rate change of the stocks during the 3 scenarios. The formulas areas follows:

$$\text{Variance} = \sigma_P^2 = w_s^2\sigma_s^2 + w_b^2\sigma_b^2 + 2w_s\sigma_s w_b\sigma_b\rho$$

$$\text{Standard Deviation} = \sigma_P = \sqrt{(w_s^2\sigma_s^2 + w_b^2\sigma_b^2 + 2w_s\sigma_s w_b\sigma_b\,\rho)}$$

CHPATER 1 REVIEW:

Measuring Return and Return Expectation

Scenario Analysis Method

PORTFOLIO ANALYSIS (Asset Allocation)

Asset Allocation	Weights (W%)
Stocks (Ws) =	**60%**
Bonds (Wb) =	**40%**

(Ws * rs) + (Wb * rb)

Scenario (s)	Probability (p)	ROR % (rs)	p * rs %	Deviation for Exp. Ret. (Dev.)	Square Deviation (SD) Dev^2	p * SD
Recession (Sr)	25.0%	-1.6	-0.40	-10.32	106.50	26.63
Normal (Sn)	45.0%	10.4	4.68	1.68	2.82	1.27
Boom (Sb)	30.0%	14.8	4.44	6.08	36.97	11.09
	100.0%		**8.72** %		**Variance=**	**38.99**
					SD =	**6.24** %

Figure 1.5

Chapter 2:

Covariance, Correlation and Efficient Frontiers

Covariance and Correlation Overview

- After reviewing the risk and return independently for each asset, the next step for an investor is to analyze a potential portfolio that combines all asset classes and apply the same measurement of return and calculation of risk.
- The risk and return of the combined portfolio can be significantly different when compared to each individual asset calculated separately.
- The covariance and correlation calculation represent the relationship of the risk between two or more asset classes.
- This an important step towards building a combined portfolio that achieves efficiency.

Investment Return and Risk Efficiency

- **The investment thesis is based on the idea that by diversifying or allocating your investments to various assets classes you can achieve higher efficiency**

Let's compare two asset classes such as stocks and bonds, as illustrated in figure 2.1. After we calculate each deviation from their respective mean for each economic scenario and apply the probability, we will then calculate the covariance (Cov) (Rs, Rb) for stock and bonds, which is the sum of these deviations. **The correlation is calculated as follows:**

$$\rho = \frac{\mathrm{cov}(R_s R_b)}{\sigma s \cdot \sigma_b}$$

where cov *is the covariance of the combined portfolio of returns over the standard deviation of each asset class*

The Impact of Correlation to Portfolio Efficiency: Achieving Minimum Variance

When combining two asset classes in one portfolio, the combined return, variance, and standard deviation can be achieved as follows:

Mean return (average return): $R_\rho = (w_s \cdot R_s) + (w_b \cdot R_b)$

where R_ρ is the return of the combined portfolio, R_s is the return of the stock portfolio, R_b is the return of the bond portfolio, and w_s and w_b are the percentage weights of stock and bonds, respectively.

Variance and standard deviation:

$$\sigma_P^2 = w_s^2\sigma_s^2 + w_b^2\sigma_b^2 + 2w_s\sigma_s w_b\sigma_b\rho$$

$$\sigma_P = \sqrt{(}w_s^2\sigma_s^2 + w_b^2\sigma_b^2 + 2w_s\sigma_s w_b\sigma_b\,\rho)$$

where σ_P^2 is the variance of the combined portfolio, w_s and w_b are the percentage weights of stocks and bonds, respectively, σ_s and σ_b are the standard deviation of the stocks and bonds, respectively, and ρ is the correlation.

EFFICIENCY THROUGH CORRELATION

SCENARIO PERFROMANCE ANALYSIS

Scenario (S)	Probability (p)	Stocks (s): ROR % (r_s)	p * r_s %	Deviation for Exp. Ret. (Dev.)	Square Deviation (SD) Dev^2	p * SD	Bonds (b): ROR % (r_b)	p * r_b %	Deviation for Exp. Ret. (Dev.)	Square Deviation (SD) Dev^2	p * SD
Recession (Sr)	25.0%	-12.00	-3.00	-23.70	561.69	140.42	14.00	3.50	9.75	95.06	23.77
Normal (Sn)	45.0%	14.00	6.30	2.30	5.29	2.38	5.00	2.25	0.75	0.56	0.25
Boom (Sb)	30.0%	28.00	8.40	16.30	265.69	79.71	-5.00	-1.50	-9.25	85.56	25.67
	100.0%		11.70 %		Variance=	222.51		4.25 %		Variance=	49.69
					SD =	14.92 %				SD =	7.05 %

PORTFOLIO ANALYSIS (Asset Allocation)

Asset Allocation	Weights (W%)
Stocks (Ws) =	60%
Bonds (Wb) =	40%

(Ws * r_s) + (Wb * r_b)

Scenario (S)	Probability (p)	ROR % (r_s)	p * r_s %	Deviation for Exp. Ret. (Dev.)	Square Deviation (SD) Dev^2	p * SD
Recession (Sr)	25.0%	-1.6	-0.40	-10.32	106.50	26.63
Normal (Sn)	45.0%	10.4	4.68	1.68	2.82	1.27
Boom (Sb)	30.0%	14.8	4.44	6.08	36.97	11.09
	100.0%		8.72 %		Variance=	38.99
					SD =	6.24 %

COVARIANCE & CORRELATION

Stocks (Deviation from the mean)	Bonds (Deviation from the mean)	Ds * Db	Covariance [p * (Ds*Db)]
-23.70	9.75	-231.08	-57.77
2.30	0.75	1.73	0.78
16.30	-9.25	-150.78	-45.23
		Covariance=	-102.23
		Correlation Coefficient =	-0.97

Figure 2.1

Efficient Frontier at Different Correlation Levels

The most northwestern point of the map just before the turn is the efficient frontier. This is the point with the highest possible return at the lowest possible risk, measured by the standard deviation. Figure 2.2 calculates that the lowest possible variance is estimated at 31.9% Stocks and 68.1% bonds achieving a minimum standard deviation of 1.12759 and weighted average return of 6.62655%.

FINDING RISK RETURN EFFICIENCY (EFFICIENT FRONTIER)

Portfolio A

E (rs) =	11.700
E (rb) =	4.250
σs =	14.917
σb =	7.049
Correlation=	-0.972

Minimum Variance - Efficient Frontier

W% Stock	W% bond	σ %	E(r) %
31.9%	68.1%	1.12759	6.62655

			PORTF. A CORRELATION	
Correlation =			-0.972	
Portfolio Weights			Risk	Return
W% stocks	W% bonds		σ %	E(r) %
0%	100%		7.05	4.25
10%	90%		4.91	5.00
20%	80%		2.83	5.74
30%	70%	Efficiency	1.20	6.49
40%	60%		2.10	7.23
50%	50%		4.12	7.98
60%	40%		6.24	8.72
70%	30%		8.40	9.47
80%	20%		10.57	10.21
90%	10%		12.74	10.96
100%	0%		14.92	11.70

Figure 2.2

Efficient Frontier at Different Correlation Levels

Correlations from -1 to +1

FINDING RISK RETURN EFFICIENCY (EFFICIENT FRONTIER)

Portfolio A

E (rs) =	11.700
E (rb) =	4.250
σs =	14.917
σb =	7.049
Correlation=	-0.972

			ZERO CORRELATION		POSITIVE CORRELATION		NEGATIVE CORRELATION		PORTF. A CORRELATION	
Correlation =			0.000		1.000		-1.000		-0.972	
Portfolio Weights			Risk	Return	Risk	Return	Risk	Return	Risk	Return
W% stocks	W% bonds		σ %	E(r) %	σ %	E(r) %	σ %	E(r) %	σ %	E(r) %
0%	100%		7.05	4.25	7.05	4.25	7.05	4.25	7.05	4.25
10%	90%	Efficiency	6.52	5.00	7.84	5.00	4.85	5.00	4.91	5.00
20%	80%		6.38	5.74	8.62	5.74	2.66	5.74	2.83	5.74
30%	70%		6.66	6.49	9.41	6.49	0.46	6.49	1.20	6.49
40%	60%		7.31	7.23	10.20	7.23	1.74	7.23	2.10	7.23
50%	50%		8.25	7.98	10.98	7.98	3.93	7.98	4.12	7.98
60%	40%		9.38	8.72	11.77	8.72	6.13	8.72	6.24	8.72
70%	30%		10.65	9.47	12.56	9.47	8.33	9.47	8.40	9.47
80%	20%		12.02	10.21	13.34	10.21	10.52	10.21	10.57	10.21
90%	10%		13.44	10.96	14.13	10.96	12.72	10.96	12.74	10.96
100%	0%		14.92	11.70	14.92	11.70	14.92	11.70	14.92	11.70

Figure 2.3

Efficient Frontier at Different Correlation Levels

Correlation = 0.0

Figure 2.4 shows that our portfolio (portfolio A) with an assumed zero correlation. The efficiency can be achieved around 10%–20% stock allocation, showing that the standard deviation at these levels is reduced from 7.05% (all bonds) to 6.52% at 10% stock and continues to reduce to 6.38% at 20% stock before the standard deviation increases again around 30%, showing a standard deviation of 6.66%. The lowest possible standard deviation representing the efficient frontier is calculated at 18.3% stocks and 81.7% bonds calculating a 6.637319% and 5.61335% standard deviation and combined portfolio return, respectively.

FINDING RISK RETURN EFFICIENCY (EFFICIENT FRONTIER)

Portfolio A

E (rs) =	11.700
E (rb) =	4.250
σs =	14.917
σb =	7.049
Correlation=	0.000

Minimum Variance - Efficient Frontier

W% Stock	W% bond	σ %	E(r) %
18.3%	81.7%	6.37319	5.61335

		ZERO CORRELATION	
Correlation =		0.000	
Portfolio Weights		Risk	Return
W% stocks	W% bonds	σ %	E(r) %
0%	100%	7.05	4.25
10%	90%	6.52	5.00
20%	80% Efficiency	6.38	5.74
30%	70%	6.66	6.49
40%	60%	7.31	7.23
50%	50%	8.25	7.98
60%	40%	9.38	8.72
70%	30%	10.65	9.47
80%	20%	12.02	10.21
90%	10%	13.44	10.96
100%	0%	14.92	11.70

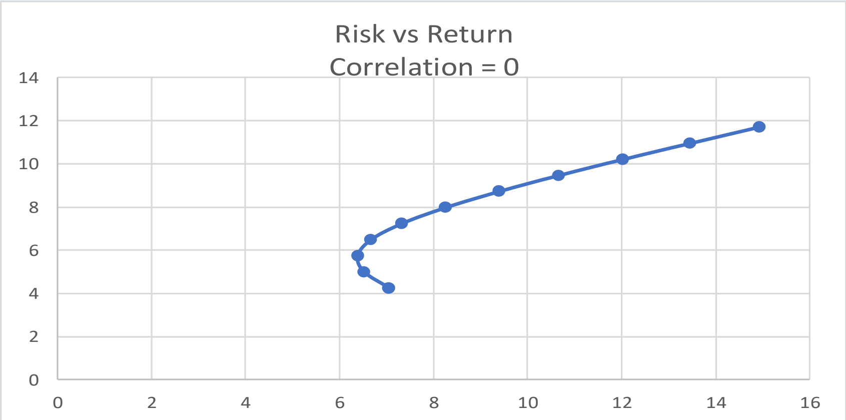

Figure 2.4

Efficient Frontier at Different Correlation Levels

Correlation = +1.0

Figure 2.5 shows our portfolio's (portfolio A) risk-versus-return allocation line assuming a perfect positive +1 correlation. At a +1 correlation there is no efficiency. As the portfolio moves from all bonds to all stock, the line is at 45-degree angle, showing that the risk continues to increase at the same pace as the portfolio manager is seeking higher returns.

FINDING RISK RETURN EFFICIENCY (EFFICIENT FRONTIER)

Portfolio A

E (rs) =	11.700
E (rb) =	4.250
σs =	14.917
σb =	7.049
Correlation=	1.000

Minimum Variance - Efficient Frontier

W% Stock	W% bond	σ %	E(r) %
0.0%	100.0%	7.04894	4.25000

		POSITIVE CORRELATION	
Correlation =		1.000	
Portfolio Weights		Risk	Return
W% stocks	W% bonds	σ %	E(r) %
0%	100%	7.05	4.25
10%	90%	7.84	5.00
20%	80%	8.62	5.74
30%	70%	9.41	6.49
40%	60%	10.20	7.23
50%	50%	10.98	7.98
60%	40%	11.77	8.72
70%	30%	12.56	9.47
80%	20%	13.34	10.21
90%	10%	14.13	10.96
100%	0%	14.92	11.70

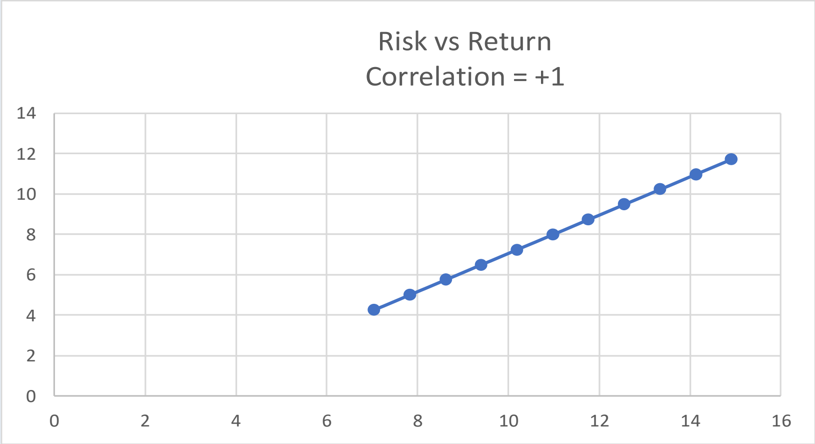

Figure 2.5

Efficient Frontier at Different Correlation Levels

Correlation = -1.0

The lowest possible standard deviation with negative -1 correlation representing the efficient frontier is calculated at 32.1% stocks and 67.9% bonds calculating a 0.00206% and 6.64145% standard deviation and combined portfolio return, respectively.

FINDING RISK RETURN EFFICIENCY (EFFICIENT FRONTIER)

Portfolio A

E (rs) =	11.700
E (rb) =	4.250
σs =	14.917
σb =	7.049
Correlation=	-1.000

Minimum Variance - Efficient Frontier

W% Stock	W% bond	σ %	E(r) %
32.1%	67.9%	0.00206	6.64145

Correlation =			NEGATIVE CORRELATION -1.00000	
Portfolio Weights			Risk	Return
W% stocks	W% bonds		σ %	E(r) %
0%	100%		7.04894	4.25000
10%	90%		4.85237	4.99500
20%	80%	Efficiency	2.65580	5.74000
30%	70%	Efficiency	0.45922	6.48500
40%	60%	Efficiency	1.73735	7.23000
50%	50%		3.93392	7.97500
60%	40%		6.13049	8.72000
70%	30%		8.32706	9.46500
80%	20%		10.52363	10.21000
90%	10%		12.72020	10.95500
100%	0%		14.91677	11.70000

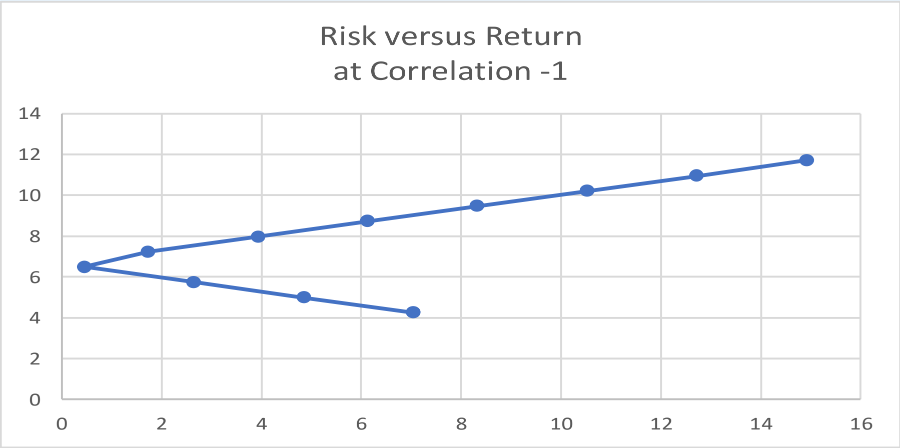

Figure 2.6

Efficient Frontier at Different Correlation Levels

Historical Analysis

HISTORICAL ANALYSIS

	Returns		Deviations from Average Return		Standard Deviation Calculation		Product from Deviation
	Stocks %	Bonds %	Stocks (Ds)	Bonds (Db)	Stocks	Bonds	Ds . Db
Year -12	-6.50	3.10	-18.18	1.02	330.33	1.03	-18.48
Year -11	-13.20	5.20	-24.88	3.12	618.77	9.71	-77.53
Year -10	-8.90	7.90	-20.58	5.82	423.33	33.83	-119.68
Year -9	25.00	6.10	13.33	4.02	177.56	16.13	53.52
Year -8	48.50	-9.50	36.83	-11.58	1356.08	134.17	-426.56
Year -7	37.60	-2.50	25.93	-4.58	672.11	21.01	-118.82
Year -6	10.50	2.50	-1.18	0.42	1.38	0.17	-0.49
Year -5	7.20	1.50	-4.48	-0.58	20.03	0.34	2.61
Year -4	-5.60	3.40	-17.28	1.32	298.43	1.73	-22.75
Year -3	17.50	-3.20	5.83	-5.28	33.93	27.91	-30.78
Year -2	21.50	3.50	9.83	1.42	96.53	2.01	13.92
Year -1	6.50	7.00	-5.18	4.92	26.78	24.17	-25.44

	Stocks	Bonds
Average Return	**11.68**	**2.08**
Standard Deviation	19.20	4.97
Covariance	-70.04	
Correlation	-0.73	

	Stocks	Bonds		
Total	**4055.24**	**272.24**		**-770.47**
Average (use n-1)	368.66	24.75	Cov=	-70.04
Standard Deviation	19.20	4.97	Correl=	-0.73

Combinced Portfolio at 30% Stocks and 70% Bonds	
Average Return	**4.96**
Standard Deviation	**3.99**

Figure 2.7

HISTORICAL ANALYSIS

Portfolio Consruction

	W% Stocks	W% Bonds	Weighted Average St. Dev.	Weighted Average Return
	0%	100%	4.97	2.08
Efficiency	10%	90%	3.34	3.04
Efficiency	20%	80%	2.86	4.00
Efficiency	30%	70%	3.99	4.96
	40%	60%	5.85	5.92
	50%	50%	7.96	6.88
	60%	40%	10.15	7.84
	70%	30%	12.39	8.80
	80%	20%	14.65	9.76
	90%	10%	16.92	10.72
	100%	0%	19.20	11.68
Efficiency	**17.8%**	**82.2%**	**2.81182**	**3.79065**

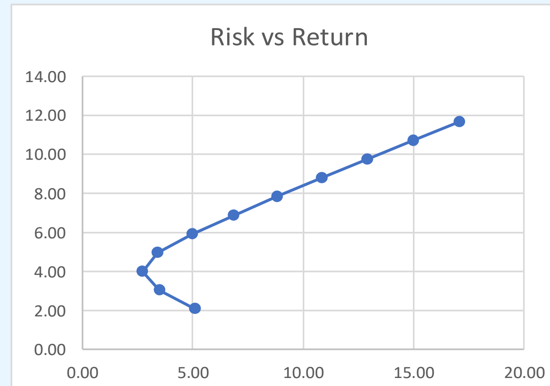

Figure 2.8

Excel formulas for average, standard deviation, covariance, and correlation:

=Average(number1, number2, . . .): highlight information range
=Stdev.p(number1, number 2, . . .) for n observations, =stdev.s for n-1 observations
=Covar(aray1, array2): highlight each comparative range
=Correl(array1, array2): highlight each comparative range

Extension to the Three-Asset Case

The question is how the investor will could improve the trade-off between risk and return by adding a new asset class in the portfolio.

THREE-ASSET CASE

Achieving efficiency by adding a third asset class

	Returns		
	Large-Cap Stocks %	**Bonds %**	**Small-Cap Stocks %**
Year -12	-6.50	3.10	-7.80
Year -11	-13.20	5.20	-16.00
Year -10	-8.90	7.90	-11.00
Year -9	25.00	6.10	21.00
Year -8	48.50	-9.50	57.00
Year -7	37.60	-2.50	49.00
Year -6	10.50	2.50	16.50
Year -5	7.20	1.50	9.00
Year -4	-5.60	3.40	-9.60
Year -3	17.50	-3.20	15.00
Year -2	21.50	3.50	27.00
Year -1	6.50	7.00	7.80
Average Return	**11.68**	**2.08**	**13.16**
Standard Deviation	**19.20**	**4.97**	**23.18**
% Holdings before Extension	30.0%	70.0%	
% Holdings including new Extension	10.0%	50.0%	40.0%

Correlation	
Large-Cap Stocks and Bonds	-0.733
Small Cap-Stocks and Large Cap Stocks	0.987
Bond and Small Cap-Stocks	-0.738

Portfolio Results	
Return for 2-Asset Holdings	4.96
Standard Deviation for 2-Asset Holdings	**3.99**
Return for 2-Asset Holdings	7.47
Standard Deviation for 3-Asset Holdings	**2.00**

Figure 2.9